

Solutions to selected questions from Sec. 9.2

Question 25

$$\begin{aligned} x + y - 5z &= -18 \\ 3x - 3y + z &= 6 \\ x + 3y - 2z &= -13 \end{aligned} ; \text{ This system has the augmented matrix } \left[\begin{array}{ccc|c} 1 & 1 & -5 & -18 \\ 3 & -3 & 1 & 6 \\ 1 & 3 & -2 & -13 \end{array} \right]$$

$$\xrightarrow{\substack{-3R_1+R_2 \\ -1R_1+R_3}} \left[\begin{array}{ccc|c} 1 & 1 & -5 & -18 \\ 0 & -6 & 16 & 60 \\ 0 & 2 & 3 & 5 \end{array} \right] \xrightarrow{\frac{1}{6}R_2} \left[\begin{array}{ccc|c} 1 & 1 & -5 & -18 \\ 0 & 1 & -\frac{8}{3} & -10 \\ 0 & 2 & 3 & 5 \end{array} \right]$$

$$\xrightarrow{\substack{-1R_2+R_1 \\ -2R_2+R_3}} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{3} & -8 \\ 0 & 1 & -\frac{8}{3} & -10 \\ 0 & 0 & \frac{25}{3} & 25 \end{array} \right] \xrightarrow{\frac{3}{25}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{3} & -8 \\ 0 & 1 & -\frac{8}{3} & -10 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\xrightarrow{\substack{\frac{7}{3}R_3+R_1 \\ \frac{8}{3}R_3+R_2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array} \right] \Rightarrow \text{Solution set: } \{(-1, -2, 3)\}$$

Question 26

$$\begin{aligned} -x + 2y + 6z &= 2 \\ 3x + 2y + 6z &= 6 \\ x + 4y - 3z &= 1 \end{aligned} ; \text{ This system has the augmented matrix } \left[\begin{array}{ccc|c} -1 & 2 & 6 & 2 \\ 3 & 2 & 6 & 6 \\ 1 & 4 & -3 & 1 \end{array} \right]$$

$$\xrightarrow{-R_1} \left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 3 & 2 & 6 & 6 \\ 1 & 4 & -3 & 1 \end{array} \right] \xrightarrow{\substack{-3R_1+R_2 \\ -1R_1+R_3}} \left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 0 & 8 & 24 & 12 \\ 0 & 6 & 3 & 3 \end{array} \right] \xrightarrow{\frac{1}{8}R_2} \left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 6 & 3 & 3 \end{array} \right]$$

$$\xrightarrow{\substack{2R_2+R_1 \\ -6R_2+R_3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 0 & -15 & -6 \end{array} \right] \xrightarrow{-\frac{1}{15}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{2}{5} \end{array} \right]$$

$$\xrightarrow{-3R_3+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{3}{10} \\ 0 & 0 & 1 & \frac{2}{5} \end{array} \right] \Rightarrow \text{Solution set: } \left\{ \left(1, \frac{3}{10}, \frac{2}{5} \right) \right\}$$

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Question 36

$$\begin{aligned} 3x + y + 3z &= 1 \\ x + 2y - z &= 2; \text{ This system has the augmented matrix } \left[\begin{array}{ccc|c} 3 & 1 & 3 & 1 \\ 1 & 2 & -1 & 2 \\ 2 & -1 & 4 & 4 \end{array} \right] \\ 2x - y + 4z &= 4 \end{aligned}$$

$$\xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 3 & 1 & 3 & 1 \\ 2 & -1 & 4 & 4 \end{array} \right] \xrightarrow{\begin{array}{l} -3R1+R2 \\ -2R1+R3 \end{array}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -5 & 6 & -5 \\ 0 & -5 & 6 & 0 \end{array} \right] \xrightarrow{-\frac{1}{5}R2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -\frac{6}{5} & 1 \\ 0 & -5 & 6 & 0 \end{array} \right]$$

$$\xrightarrow{5R2+R3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -\frac{6}{5} & 1 \\ 0 & 0 & 0 & 5 \end{array} \right]$$

The last row indicates that there is no solution $\Rightarrow$ ***Solution set is ϕ***

Question 38

$$\begin{aligned} 5x - 3y + z &= 1 \\ 2x + y - z &= 4; \text{ This system has the augmented matrix } \left[\begin{array}{ccc|c} 5 & -3 & 1 & 1 \\ 2 & 1 & -1 & 4 \end{array} \right] \end{aligned}$$

$$\xrightarrow{\frac{1}{5}R1} \left[\begin{array}{ccc|c} 1 & -\frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ 2 & 1 & -1 & 4 \end{array} \right] \xrightarrow{-2R1+R2} \left[\begin{array}{ccc|c} 1 & -\frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ 0 & \frac{11}{5} & -\frac{7}{5} & \frac{18}{5} \end{array} \right] \xrightarrow{\frac{5}{11}R2} \left[\begin{array}{ccc|c} 1 & -\frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ 0 & 1 & -\frac{7}{11} & \frac{18}{11} \end{array} \right]$$

$$\xrightarrow{\frac{3}{5}R2+R1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{2}{11} & \frac{13}{11} \\ 0 & 1 & -\frac{7}{11} & \frac{18}{11} \end{array} \right]$$

The equations that correspond to the final matrix are:

$$x - \frac{2}{11}z = \frac{13}{11} \quad \text{and} \quad y - \frac{7}{11}z = \frac{18}{11}$$

This system has infinitely many solutions. We will express the solution set with z as the arbitrary variable. Therefore,

$$x = \frac{2}{11}z + \frac{13}{11} \quad \text{and} \quad y = \frac{7}{11}z + \frac{18}{11}$$

$$\Rightarrow \text{Solution set: } \left\{ \left(\frac{2}{11}z + \frac{13}{11}, \frac{7}{11}z + \frac{18}{11}, z \right) \right\}$$